



© 1997–2004, Millennium Mathematics Project, University of Cambridge.

Permission is granted to print and copy this page on paper for non-commercial use. For other uses, including electronic redistribution, please contact us.

January 1999

Regulars

Solution to Puzzle No. 6



For the question see "[Puzzle No 6 – world cup medallions](#)" in issue 6.

Firstly, we'll suppose that there are N different medallions to collect. This makes the working easier, plus the result we shall obtain is more general; we can simply put $N=22$ at the end to obtain the answer to the problem.

As for [the hint](#), we define the *random variable* $X_n = T_n - T_{n-1}$ where T_n is the number of medallions you have collected when you *first* own n different medallions.

The next step is to try to find the *distribution* of X_n i.e. to calculate [an error occurred while processing this directive]

for different values of n and j where [an error occurred while processing this directive]

Let us now fix such an n and j . In order that $X_n = j$ we must have first picked $j-1$ medallions which we already had in our collection of $n-1$ different medallions, and then picked a new medallion. Assuming that each different type of medallion is equally likely to be picked at any stage (which we're told), then denoting p to be the probability that we pick a new medallion, and q to be the probability that we pick a medallion already in our collection, we have [an error occurred while processing this directive]

from which we see that [an error occurred while processing this directive]

Hence X_n has the *geometric* distribution with parameter p (defined as above). We may write this in symbols as [an error occurred while processing this directive]

It's not too hard to show that if a random variable [an error occurred while processing this directive]

then [an error occurred while processing this directive]

Hence [an error occurred while processing this directive]

The *average* number of medallions collected in total is therefore [an error occurred while processing this directive]

Solution to Puzzle No. 6

and, after a little rearranging, we obtain the result: [an error occurred while processing this directive]

So, putting $N=22$ into this formula we see that on average the number of medallions we need to collect to obtain the full set is about 81.



Plus is part of the family of activities in the Millennium Mathematics Project, which also includes the NRICH and MOTIVATE sites.