FINDING YOUR WAY AROUND THE ARCTIC USING GPS

The GPS satellites that orbit the Earth continually transmit messages containing the exact time of transmission, and the exact location of the satellite at the time of transmission. When a GPS receiver receives these messages, it uses the time delay to work out exactly how far away it is from the satellite.

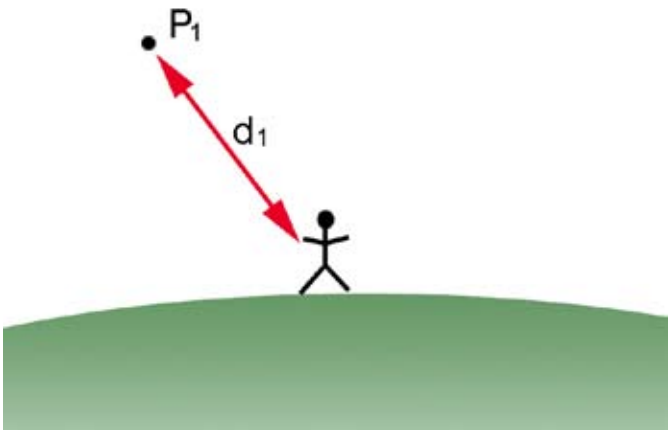
Ann checking her GPS position before setting off in the morning

Question 1: The signal transmitted by a satellite travels at the speed of light, which is approximately 300,000 km/s. The ice team's GPS receiver receives a signal from a satellite and calculates that the time the signal took to reach the receiver was 0.07 seconds. How far is the receiver from the satellite?

"Without GPS technology, the ice team would have to navigate by the stars!"

Simon Harris-Ward
Operations
Director

Question 2: Suppose the receiver has calculated that it is a distance d_1 away from a particular satellite (for simplicity, imagine that the satellite is stationary at point P_1). In the plane containing the satellite and the receiver, what kind of shape is formed by all the points that lie at distance d_1 from the satellite?



A two-dimensional view

Question 3: Is one satellite enough to calculate the explorers' exact location?

Question 4a: In a two-dimensional coordinate system mark the points P_1 and P_2 with coordinates (0,20) and (30,20) respectively. These denote the locations of two satellites that lie in a plane with the explorers (one unit in the coordinate system corresponds to 1000 km in our two-dimensional world, ground level lies on the x-axis). The receiver has calculated that the explorers are exactly 25 units (25,000km) away from each of the two satellites. Can you find the exact location of the explorers by drawing circles centred on the two satellites?

Question 4b (harder): In a two-dimensional coordinate system mark the P_1 and P_2 with coordinates (0,20) and (30,20) respectively. These denote the locations of two satellites that lie in a plane with the explorers (one unit in the coordinate system corresponds to 1000 km in our two-dimensonal world, ground level lies on the x-axis). The receiver has calculated that the explorers are exactly 25 units (25,000km) away from each of the two satellites. Can you calculate the exact location of the explorers by using the equations of the circles centred on the two satellites?

Question 5: **Now consider the more general case, when the satellites and the explorers do not all lie in a plane. In three dimensions, what kind of shape is formed by all the points at distance d from the satellite at point P? Are two satellites enough to calculate the exact location of the explorers in three dimensions? How do questions 2 and 3 generalise to three dimensions?**

Question 6a (challenging): **Write down the equations of the sphere centred at the point (0,15,0) with radius 25, and the sphere centred at (0,-15,0) with radius 25. Observing the symmetry, what can you say about the circle of intersection?**

Question 6b (challenging): **What are the centre and radius of the circle C of intersection?**

Question 6c (challenging): **Observing the relative positions of the circle C of intersection and the sphere centred at the point (0,0,40) with radius 20, find the point(s) where the sphere and the circle meet.**

Conclusion: GPS systems are based on this process, which is called *trilateration*. However, to minimise errors and to gain extra information, real GPS systems calculate the distance between the receiver and four satellites.