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Features



Euler's polyhedron formula

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No simple polyhedron has seven edges.

Proof: We show first that for any polyhedron we have $2E \geq 3F$ and $2E \geq 3V$. The faces of the polyhedron are polygons, each bounded by a number of sides. Along each edge exactly two faces come together, so an edge corresponds to exactly two sides: the total number of sides is $2E$. We also notice that any face has *at least 3* sides, so the total number of sides is *at least 3* times the number of faces. Thus we get:

The total number of sides = $2E$

and

The total number of sides $\geq 3F$.

Putting this together we get:

$$2E \geq 3F,$$

proving our first inequality.

To prove the second inequality we count the total number of *ends* of edges. Each edge has two ends, so the total number of ends equals $2E$. At each vertex at least three edges come together, so the total number of ends of edges is at least 3 times the number of vertices. Putting this together we get:

Euler's polyhedron formula

$2E = 3V$.

Now, if a polyhedron has 7 edges, then $3F = 14$ and $3V = 14$. This means that both F and V cannot be bigger than 4. A little thought will convince you that every polyhedron has strictly more than three faces, so we must have $F=4$. Similarly we get that $V=4$. This gives

$$V - E + F = 4 - 7 + 4 = 1 \neq 2.$$

This tells us that our hypothetical seven-edged polyhedron cannot exist, for if it did, then Euler's formula would hold and the above sum would have to be equal to 2 QED!

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