March 2005
Regulars

Puzzle page

Party people

Is it possible to hold a party at which no two people have the same number of friends as each other?

The solution

The answer is no, it is not. The proof is a so−called "proof by contradiction": suppose the opposite and show that this inexorably leads to a contradiction.

So we start by supposing that we have a party, with N people attending, and that no two of those people know the same number of people as each other. Since the most friends anyone can have at the party is $N-1$, and the fewest is 0, there must be exactly one person with each number of friends between 0 and $N-1$ inclusive.

But the person with $N-1$ friends knows everybody at the party − including the person with no friends! This is the contradiction, and it implies that the assumption wasn't true: there cannot be such a party.

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